Electrical Equipment - Course PI 30.2

IMPEDANCE

OBJECTIVES

On completion of this module the student will be able to:

1. In a few words, define
   a) Capacitive and inductive reactance
   b) Impedance

2. State, in writing, that:
   a) For a capacitor fed from an ac source the current leads the voltage by 90°
   b) For an inductor fed from an ac source the current lags the voltage by 90°

3. Explain, in writing, how a circuit can be:
   a) Inductive
   b) Capacitive

4. Given the circuit component values and the type of connection, identify the circuit behaviour as inductive or capacitive.

5. Define in one sentence the term "Power Factor Angle".

1.0 INTRODUCTION

This module introduces the student to the concept of:

(a) impedance.

(b) inductive and capacitive behaviour of an ac circuit.

(c) power factor angle.

2.0 RESISTANCE, REACTANCE AND IMPEDANCE

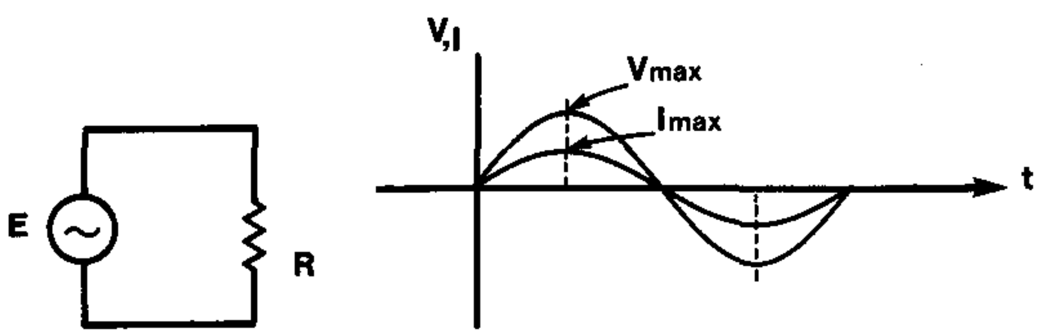
Resistance and the reactance contribute to the impedance. It is therefore necessary to have an understanding of the behaviour of these components when an ac voltage is applied to a circuit containing resistance and reactance.

2.1 Resistance

When a resistor is supplied with an ac voltage the current flowing through it will be a sine wave whose maximum value depends on the resistance R. Resistance is independent of frequency. Response of the resistance to the increase in voltage across it is the rise of current simultaneously. When the voltage decreases the current will also decrease at the same time. This phenomenon in technical terms is stated as follows:

The Voltage drop across a resistor is in phase with the current flowing through it.

This is also shown diagramatically in Figure 1.



Voltage and Current Relationship in a Resistor

Figure 1

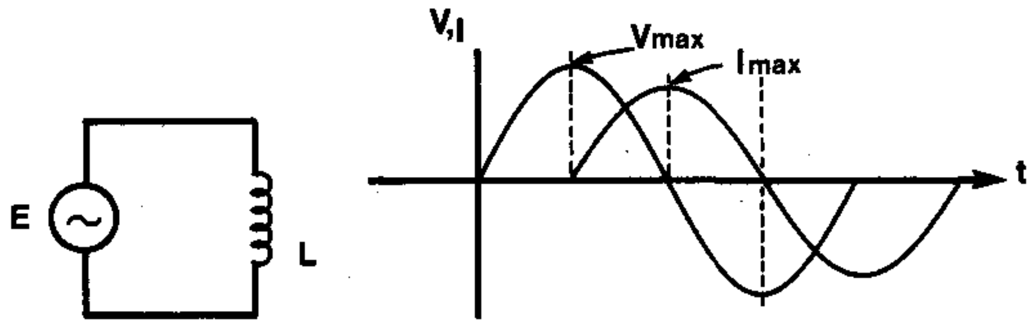
2.2 Reactance

Reactance in the circuit is contributed by the two reactive components, ie, inductance and capacitance. Inductive and capacitive reactances are calculated by the following expressions as discussed in PI 26.32-1 and 33-1.

$$X_L = 2\pi fL$$

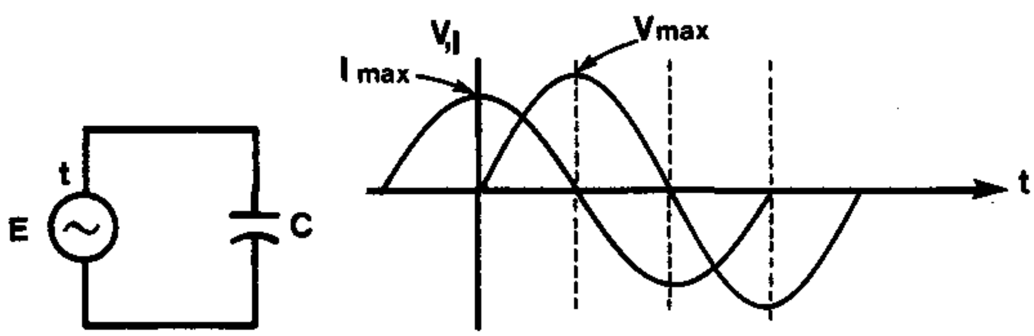
$$X_C = \frac{1}{2\pi fC}$$

The opposition offered by the inductor or a capacitor is called reactance because the inductor reacts to the change in current and the capacitor reacts to the change in voltage. Current and voltage relationships in a capacitance and an inductance are shown in Figures 2(a) and (b).



Inductor: Current Lags the Voltage by 90°.

Figure 2(a)



Capacitor: Current Leads the Voltage by 90°.

Figure 2(b)

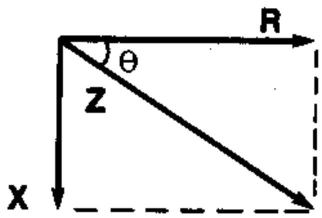
Examining the expressions for X_L and X_C, it is also apparent that the increase in frequency will increase X_L while X_C will decrease, and a decrease in frequency will cause X_L to decrease but X_C will increase. This indicates that the two reactances effectively function in opposition to each other. The opposing characteristics of X_L and X_C as well as the voltage and current relationships can be displayed by the use of vectors. (See Appendix.)

2.3 Impedance

A combined opposition effect to current flow of RL, RC or RLC is called impedance. Impedance is represented by Z and its unit is the ohm. It is calculated by using the following expression.

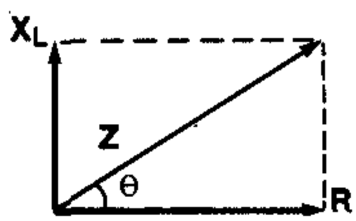
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \text{Arc tan} \left(\frac{X_L - X_C}{R} \right)$$



RC Circuit

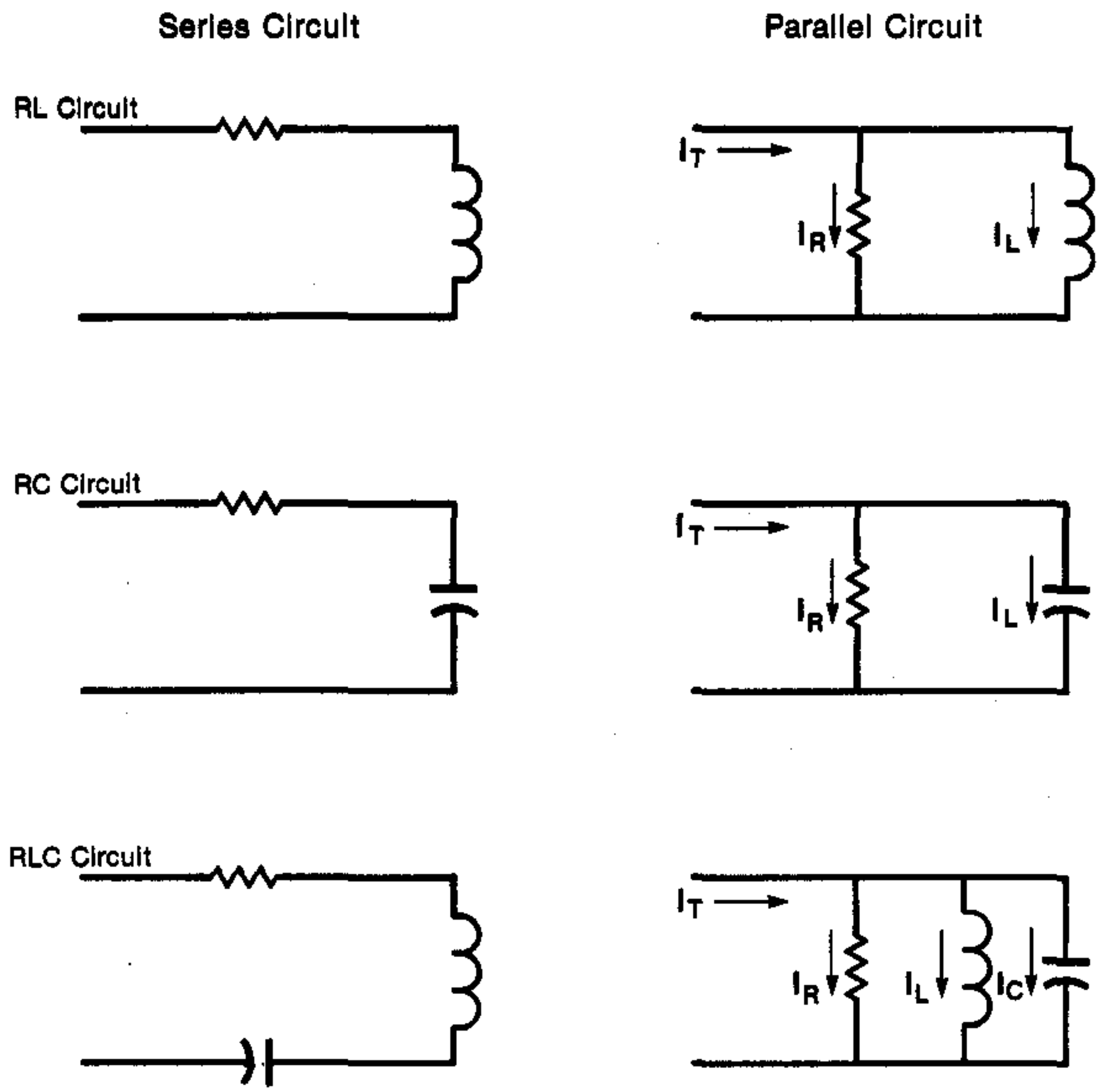
Figure 3(a)



RL Circuit

Figure 3(b)

A RL, RC or RLC circuit can be connected in series or in parallel, see Figure 4.



Series and Parallel Circuits

Figure 4

3.0 CIRCUIT BEHAVIOUR

Overall behaviour of a circuit can be inductive or capacitive depending on the following two factors.

(a) Relative magnitudes of X_L and X_C.

(b) Series or parallel connection.

3.1 Inductive Behaviour

An ac circuit under the following conditions will behave inductively, **ie, total current in the circuit will lag the applied voltage by an angle θ.**

(a) A series circuit composed of resistance and inductance.

(b) A series circuit composed of R, L and C but $X_L > X_C$.

(c) A parallel circuit composed of R, L.

(d) A parallel circuit composed of R, L and C but $X_L < X_C$.

The angle of lag between the source voltage and the total current will depend on the relative magnitudes of reactance and resistance as given below.

Do not memorize the following expressions.

$$\theta = \text{Arc tan } \frac{X_L}{R} \quad \text{for series RL circuit.}$$

$$\theta = \text{Arc tan } \frac{X_L - X_C}{R} \quad \text{for series R, L \& C circuit.}$$

$$\theta = \text{Arc tan } \frac{R}{X} \quad \text{for parallel RL circuit.}$$

$$\theta = \text{Arc tan } \frac{R(X_C - X_L)}{X_C \cdot X_L} \quad \text{for parallel R, L \& C circuit.}$$

This angle θ between the source voltage and the total current is given the name power factor angle. Its importance will be discussed in the lesson PI 26.36-2.

3.2 Capacitive Behaviour

An ac circuit under the following conditions will behave capactive, ie, **the total current will lead the applied voltage by an angle θ.**

(a) A series circuit composed of resistance and capacitance.

(b) A series circuit composed of R, L and C but $X_C > X_L$.

(c) A parallel circuit composed of R and C.

(d) A parallel circuit composed of R, L and C but $X_C < X_L$.

The angle of lead between the total current and the source voltage will depend on the relative magnitudes of reactance and resistance as given below.

Do not memorize the following expressions.

$\theta = \text{Arc tan} \dfrac{X_C}{R}$ for series RC circuit.

$\theta = \text{Arc tan} \dfrac{R}{X_C}$ for parallel RC circuit.

$\theta = \text{Arc tan} \dfrac{X_C - X_L}{R}$ for series R, L and C circuit.

$\theta = \text{Arc tan} \dfrac{R(X_C - X_L)}{X_C \cdot X_L}$ for parallel R, L and C circuit.

Again the angle θ is referred to as power factor angle and will be discussed in the lesson PI 26.36-2.

ASSIGNMENT

1. Define the term "impedance". Indicate how it is represented in the formula and what is its unit. (Section 2.3)

2. What is meant by the inductive behaviour or the capacitive behaviour of a circuit in terms of voltage and current relationships. (Section 3.1, 3.2)

3. Identify an inductive or capacitive circuit in the following cases. (Section 3.1, 3.2) See answers, Page 12.

   (a) A series circuit $R = 10\ \Omega$, $X_L = 20\ \Omega$, $X_C = 10\ \Omega$.

   (b) A parallel circuit $R = 10\ \Omega$, $X_L = 20\ \Omega$, $X_C = 10\ \Omega$.

   (c) A series circuit $R = 5\ \Omega$, $X_L = 1\ \Omega$.

(d) A series circuit $R = 20\,\Omega$, $X_C = 20\,\Omega$.

(e) A parallel circuit $R = 7\,\Omega$, $X_L = 12\,\Omega$, $X_C = 24\,\Omega$.

(f) A parallel circuit $R = 100\,\Omega$, $X_C = 50\,\Omega$.

(g) A parallel circuit $R = 75\,\Omega$, $X_L = 100\,\Omega$.

4. What is power factor angle?

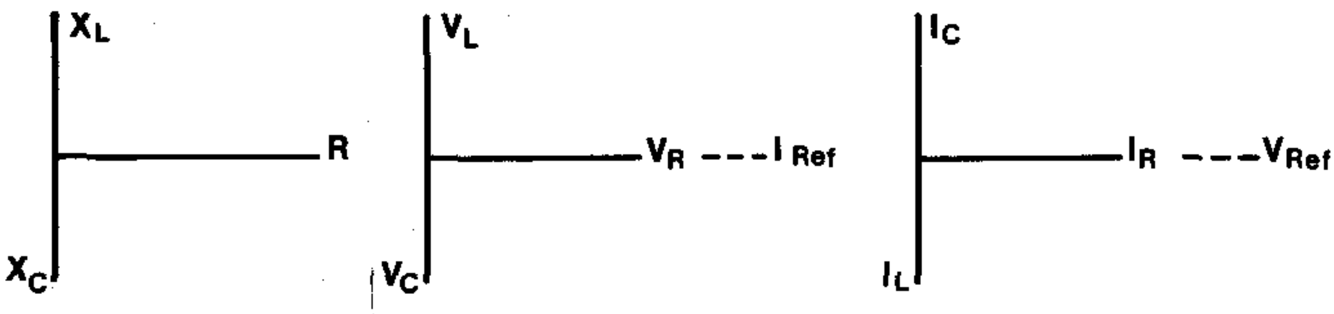
NOTE: Additional assignments are available on computer. If you wish further practice, ask the Course Manager for the mini disk package "Single-Phase Circuit Simulation" (3 disks).

S. Rizvi

APPENDIX

VECTORIAL NOTATION

The reactive phenomenon of L and C, their opposing characteristics and the voltage and current relationships can be represented conveniently by the use of vector diagrams shown in Figure 1. In all cases resistance or the voltage across it or the current through it is taken as the reference. Counter-clock rotation of vector is considered as positive direction.



Resistance and Reactances (a) | Voltages (b) | Currents (c)

Vectorial Representation

Figure 1

Examine Figure 1(a). Horizontal axis is assigned to the resistance and used as the reference.

Positive vertical axis is assigned to X_L }
Negative vertical axis is assigned to X_C } opposing characteristics

Examine Figure 1(b) used in series circuits. Current being same in the series circuit it is used as reference. Horizontal axis is assigned to the voltage across the resistance.

Positive vertical axis is assigned to the voltage across the inductance because the voltage across the inductance leads by 90° as compared to the current.

Negative vertical axis is assigned to the V_C because the voltage across a capacitor lags by 90° as compared to the current. This representation is useful in series circuits since the current in a series circuit is the same through each component.

Examine Figure 1(c) used in parallel circuits. Voltage being the same in parallel circuit, it is used as reference.

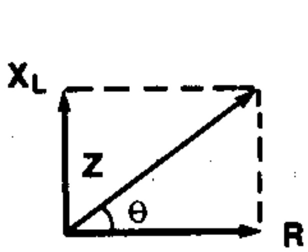
Horizontal axis is assigned to the current through the resistance.

Positive vertical axis is assigned to the current through the capacitance. This indicates that the current in a capacitor leads the voltage across it by 90°.

Negative vertical axis is assigned to the current through the inductor. This indicates the fact that the current in an inductor lags the voltage across it by 90°.

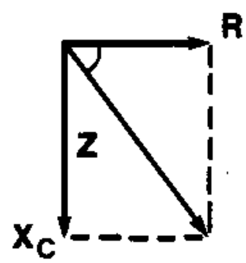
Impedance

Impedance can be determined vectorically by adding the resistance and the reactance vectors as shown in Figure 2.



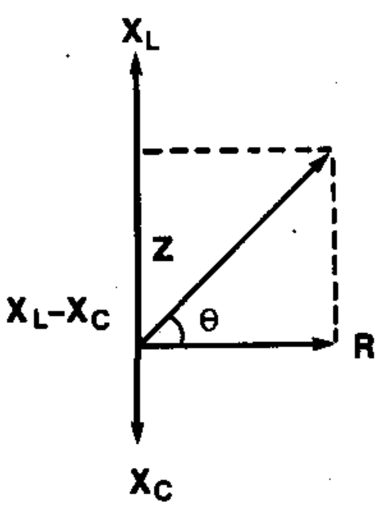
R, L Circuit

(a)



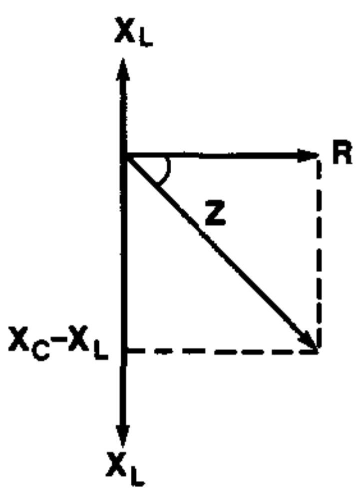
R, C Circuit

(b)



R, L, C Circuit $X_L > X_C$

(c)



R, L, C Circuit $X_C > X_L$

(d)

Answers For Assignment Question 3

(a) $X_L > X_C$ in a series circuit: circuit is inductive.

(b) $X_C < X_L$ in a parallel circuit: circuit is capacitive.

(c) Series R, L circuit: circuit is inductive.

(d) Series R, C circuit: circuit is capacitive.

(e) $X_L < X_C$ in a parallel circuit: circuit is inductive.

(f) R, C parallel circuit: circuit is capacitive.

(g) R, L parallel circuit: circuit is inductive.